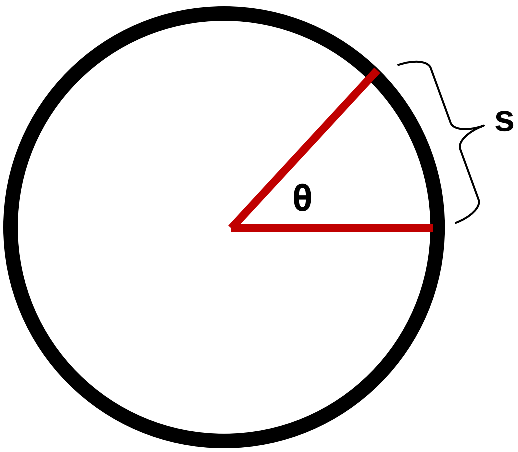
Rotational Kinematics and Dynamics: Variables

Below is a diagram of a circle, with some things labeled. ***r*** is the radius, ***θ*** is an angle, and ***s*** is the arc length between the two radii. We are going to write down a few things about circles together in groups and as a class.



1. Write down the following for a circle:

Circumference:

Area:

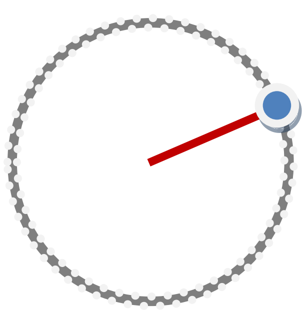
What is the arc length for a complete circle? (s = ?)

What is the angle for a complete circle in degrees? In *radians*?

2. Using your equation for arc length and your answer for the angle in radians, write an equation relating s, r and θ.

Angular velocity

We've already seen things in uniform circular motion and how that relates to the speed that the object is going. If I have a ball going around in a circle, draw the velocity vector for the ball at the point where the ball is.



3. How does the velocity (geometrically) compare to the circle that is the ball's path?

4. Write down the expression for centripetal acceleration in terms of this velocity:

If we think about a rigid object (one that doesn't bend or flex or have separate moving parts) such as a CD, the whole thing goes around together. The drawing has three different points on it.

5. If point A goes around 360°, how many degrees do points B and C go around?

6. If point C takes a time of Δt, how much time do points B and A take to go around? (The time it takes to go around once is called the *Period (T)*)

7. Compare the *tangential velocity* for points A, B, and C.

However, we can define an angular velocity (ω) which is the change in degrees per change in time that it takes to go around. $\omega = \Delta\theta/\Delta t$.

8. Based on this, compare the *angular velocity* for points A, B and C.

We can write similar things for rotation as for linear motion. We have position (θ), velocity (ω), and acceleration (α). Given what you know so far, can you guess what angular acceleration would be? *(Hint: Think of how we first defined acceleration in terms of linear kinematics)*

Great! Let's fill in the table:

	Linear	**Angular**
Position		
Velocity		
Acceleration		

Note: The angular variables are still vector quantities! We define + or – based on clockwise or counterclockwise. Just as with regular kinematics it doesn't matter as long as you are consistent in your choice within a problem. The book uses counterclockwise (CCW) as positive (+) and clockwise (CW) as negative (-).

<u>Relationships between Linear and Angular variables</u>

All right, that's all well and good. But sometimes we need to convert between angular and linear variables. Let's see if we can't come up with some relationships.

9. Let's use the variable v_T for tangential velocity, so we're explicit on what velocity we're using. For part of a circle, what is that distance traveled called? Write down the tangential velocity in terms of distance over time for part of a circle:

10. Using your result from question 2, substitute for the arc length so that you have an expression for v_T in terms of the angle traveled and the radius.

11. Now you (hopefully) have something involving $\Delta\theta/\Delta t$. Look back at your table and replace this with the appropriate angular quantity:

12. This is the relationship between tangential velocity and angular velocity! Notice you have an "r" in there. Do the units work out? Explain, noting that radians aren't really a unit (they're considered *unitless*). Can you use this equation if θ is in degrees?

13. Now let's relate tangential acceleration to angular acceleration. Again being explicit, write an equation for tangential acceleration (a_T) in terms of tangential velocity (v_T). *(Hint: remember this is just your regular linear acceleration and velocity like we've always been using!)*

14. Now use your expression from question 11 to substitute for v_T.

15. For a rigid object, does r change? Write your expression from 14 in terms of what *does* change. Then look up at your table and relate it to an angular variable.

Congratulations! You have related your angular variables to your linear variables!

16. You have an angular acceleration. This means that our disc is speeding up or slowing down. Do we have uniform circular motion?

Kinematic Equations: Linear vs. Angular

17. Write down our two linear kinematic equations (from way back when). Be sure to use vector notation!

18. *By analogy*, write down two similar equations for angular motion:

Hooray, you've derived half of Chapter 7! Go you!